DOCUMENT RESUME

ED 245 723 JC 840 337

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TITLE Calculators Do More Than Compute.
PUB DATE Jul 84
NOTE 16p.; Paper presented at the Sloan Foundation Conference on New Directions in Two-Year College Mathematics (Atherton, CA, July 11-14, 1984).
PUB TYPE Speeches/Conference Papers (150) -- Reports - Descriptive (141)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Calculators; *Mathematics Instruction; Postsecondary Education; *Remedial Mathematics; Secondary Education; *Teaching Methods
IDENTIFIERS *Pre Algebra

ABSTRACT

Drawing from experiences at Ohio State University (OSU) with remedial college students and with 12th grade students without elementary algebra skills, this paper outlines a numerical approach to algebra instruction using handheld calculators. Introductory material discusses problems with traditional approaches to math education and outlines some instructional developments made possible through the use of calculators. Next, background is presented on the experimental use of handheld calculators at OSU, including information on students' skill and achievement levels and the rationale for using concrete problem situations, rather than axioms and algorithms, and calculators, rather than pencil and paper. Then, several features of the numerical, problem-solving approach used at OSU are explored: (1) students with calculators can effectively use numerical methods to solve problems they are unable to solve algebraically; (2) using calculators, students can encounter concepts and relationships in problem situations before the concepts are formalized; (3) investigating problems in many special cases permits variables to be introduced in a natural way to describe the general case; (4) calculators require students to pay attention to arithmetic ideas that would otherwise be of little interest to them; and (5) calculators permit the early introduction of graphing. Finally, further developments of this approach are illustrated with regard to strengthening the pre-algebra component of 7th and 8th grade courses. (HB)

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Atherton, California

July 11-14, 1984

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CALCULATORS DO MORE THAN COMPUTE

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SUMMARY

This paper has been prepared for the June, 1984 Sloan-sponsored conference, New Directions in Two-Year College Mathematics. It describes changes in the approach to algebra and geometry that have resulted from using handheld calculators with remedial students at Ohio State and with twelth grade students who have no skills in elementary algebra.

The approach described is highly numerical and depends on concrete problem situations to suggest key concepts of algebra. Calculators are used to give students greater confidence, to get them past arithmetic difficulties and into significant mathematics more quicky, and to enable the investigation of realistic problems. More importantly, calculators provide an entrance to mathematics quite different from traditional approaches. Several features of this numerical approach are discussed:

-- using numerical methods to solve problems that students are not able to solve algebraically

-- encountering basic concepts and relationships in problem settings before the concepts are formalized

-- investigating problems in many special cases so that variables can be introduced in a natural way to describe the general case

-- using calculators to draw students' attention to arithmetic ideas that otherwise would be of little interest to them

-- using calculators to permit the early introduction of graphing so that graphing can provide a bridge between numerical work and the formalism of algebra.

CALCULATORS DO MORE THAN COMPUTE

Joan R. Leitzel

Over the last several years there has been considerable experimentation with different types of delivery systems for traditional college mathematics content. We know about Keller plans, modified Keller plans, computer assisted instruction, computer managed instruction, mathematics laboratories, mathematics workshops, peer tutoring, video-tape support, mathematics modules, self-pacing, retest options, and so on. Many departments feel these developments have been useful in improving their instruction of traditional content. However, only recently have questions begun to be raised about the appropriateness of that content, the way the mathematical ideas are approached, and the order in which topics are sequenced.

It is good that these questions can be raised with respect to the curriculum of two-year colleges. Beginning university courses continue to be constrained by the assumption that all paths lead one way or another to calculus. Two-year colleges, however, admit to more than one mathematical goal for students and hence provide an opportunity for some creative re-evaluation of course content and approach. Not all such questions will be answered by hard thinking. Some experimentation must be done, and two-year colleges can be an important part of that experimentation.

At least two phenomena press us to take seriously the need to reevaluate what we are doing. One is the evidence that large numbers of students are not learning mathematics well in our college preparatory programs. For this reason, as we seek better ways, we must also attempt to understand what causes

present programs to be ineffective. A second phenomenon, of course, is the availability of low cost calculators and computers. These we sense should give new ways of communicating mathematics and even open up new topics for our students.

My colleagues and I have had several years experience using calculators in mathematics instruction at Ohio State. Calculators were first introduced in the remedial sequence in 1974. Now scientific calculators (with algebraic logic and hierarchy) are required in all courses below calculus, and one version of the calculus sequence uses computers and programmable calculators. The mathematics placement exam has been adjusted so that students may use calculators on it if they wish.

In this paper I want to discuss briefly the effect that using calculators has had on our teaching of mathematics to college-age students who have serious deficiencies in mathematics. We have taught many of these students as freshmen at the University, and in the last couple years we have worked with them as high school juniors and seniors. We have been limited in this course development by the goal of preparing students for standard precalculus courses. Our efforts have been less with defining alternative course content than with finding alternative ways of approaching traditional mathematics to make it accessible to larger numbers of students. It is the change of approach made possible by calculators that I want to describe in this paper. Hopefully, what we have learned about the potential for calculators in the teaching of algebra and geometry will be useful to those who are thinking about other mathematics courses at the beginning level.

What We Think We Know

In the academic year 1983-84, approximately 60,000 juniors in 614 Ohio high schools were tested through the Early Mathematics Testing Program. Almost all of these students said they planned to attend two-year or four-year colleges. Forty percent of them tested at the lowest level, "level 5" by our designation. Among level 5 students only about 5% can correctly interpret $\frac{x^6}{x^2}$ (although 40% can evaluate $\frac{10^6}{10^2}$); fewer than 5% can choose from a multiple choice list a solution to a linear system of equations with integer coefficients. Thus we say that level 5 students have no usable understanding of elementary algebra. However, 90% of these students have successfully completed Algebra I in their schools and more than 50% were enrolled in Algebra II when tested in the winter or spring of their junior year. This is the bad news.

The good news is we have evidence that a high percentage of these students can be successful with this mathematics when the approach to it is changed. The approach we have come to use is a highly numerical one with key concepts suggested in concrete problem situations. This approach is possible because of the availability of handheld calculators.

At Ohio State approximately 75% of the students who move from the remedial sequence into university level courses are successful in those courses. Although some of us might hope that this number could be higher, it is comparable to the success rates in the mathematics sequences at higher levels and is far better than when we taught arithmetic, elementary algebra, and intermediate algebra separately for this audience.

Of course, there are many reasons why students who are unsuccessful in high school may be successful in college. But we have demonstrated that students who test at level 5 as high school juniors can still learn much of the mathematics they need for college entry in their senior year. In 1981-82, we piloted an alternative course for high school seniors who had placed at the lowest levels on the junior year exam. The materials for this course were field tested in 1982-83 in 41 Ohio high schools with more than 1000 students; at the beginning of the year, 750 of these students were at level 5, 140 at level 4, and 110 above level 4. (In Ohio level 3 is an acceptable university entry level for non-science students.) Over 80% of the level 5 students finished the course above level 5 with almost 70% above level 4; 91% of the level 4 students finished above level 4. Two-hundred of these students entered Ohio State in autumn 1983. Their mathematics grades during the academic year 1983-84 matched closely the grade distributions of the courses they were in.

The senior year high school course, like our university remedial sequence, takes a highly numerical approach to the traditional content of algebra and geometry. In developing these courses we have learned that a calculator can do much more than compute. It can provide access to many key concepts of algebra and permit the numerical investigation of problem scenarios that otherwise would be inaccessible to beginning students.

We are convinced that there are better approaches with college-age students than starting with arithmetic algorithms and moving again through grades 5, 7, 9, and 11. This approach did not work for these students the first time. It suggests that students cannot do interesting mathematics until they can compute accurately with, for example, fractions. In algebra it requires students very early to use formal notation, to reason from axioms, and

to handle greater abstraction than they are able to handle. It does not make use of their abilities to do mathematics within concrete problem situations. With calculators we can give up the fetish with paper and pencil computational skills and the formal, axiomatic approaches to elementary algebra. We can start with problems and calculators rather than with algorithms or axioms.

Using Calculators to Change the Approach

Our initial purpose in giving calculators to remedial students in 1974 was to get around arithmetic drill and practice, and to get students with weak computational skills into significant mathematics more quickly. There were other benefits. We immediately observed that calculators gave students more confidence and also enabled us to write more realistic and more demanding problems. These results were perhaps not surprising. Less predictable was the extent to which calculators provided an entrance to mathematics quite different from traditional approaches. Some features of this numerical, problem-solving approach are described below.

* Students with calculators can effectively use numerical methods (i.e. successive approximations or guess-and-check techniques) to solve problems that they are not able to solve algebraically.

With calculators, students are able to investigate problem situations in many special cases. Rather than merely attempting to answer a single question in isolation, students can investigate the relationships between quantities in a problem in many special cases, record information, and on the basis of this information answer many additional questions.

EXAMPLE

Water is added to 40 gallons of a 25% alcohol solution. Complete this chart to show how the concentration of the solution depends on the amount of water added.

Amount of water added (gal.)	Amount of new solution (gal.)	Amount of alcohol in new solution (gal.)	% alcohol in new solution
5	40 + 5 = 45	.25(40) = 10	$\frac{10}{45} \doteq 22.2\%$
10			
20			
			12.5%

How many gallons of water must be added to 40 gallons of a 25% salt solution to give a 12.5% solution? How many gallons must be added to give a concentration less than 12.5%? How many gallons must be added to give a 10% solution?

Students become skilled in this type of problem solving and come to believe that with a calculator they can begin to solve most problems. Problems need not be limited to those that can be described by linear or quadratic equations. With calculators beginning students can solve many problems they could not approach algebraically.

* Students using calculators can encounter basic concepts and relationships in problem situations before the concepts are formalized.

For example, in a problem about the growth of a bacterial culture where the doubling period is given as 10 hours, it is natural to ask for the number of bacteria after one hour. Initially, $2^{1/10}$ is a number the calculator gives a value for. Students can compute with fractional exponents before they deal conceptually with $a^{1/n}$.

The concept of function is a fundamental concept in algebra. If students only solve a single exercise in a given problem situation, they do not sense the underlying functional relationships in the problem. However, when they compute many cases, they do see the dependence of one quantity on another (i.e., in the first example that the concentration depends on the amount of water added to the solution). Calculators permit the extensive computation required to display these relationships.

Since algebra generalizes arithmetic, most algebraic concepts can be anticipated in numerical settings. This experience is particularly important for students who are not strong in mathematics.

* Investigating problems in many special cases permits variables to be introduced in a natural way to describe the general case.

The concept of variable is another absolutely fundamental concept of algebra. Our experience is that students who take algebra courses and two years later know no algebra are students who have little understanding of what a variable is or why it is introduced. If problem settings are investigated

numerically in many cases, then introducing a variable to represent a set of numbers is meaningful.

EXAMPLE

Civil service employees anticipate a 7% raise.
Complete the following chart.

Present salary ($)	12,000	15,500	22,000	25,400	x
New salary ($)					

Because students can solve problems numerically using calculators and guess-and-check procedures, the formalism of writing and solving equations can be delayed until extensive numerical experience supports these procedures. Thus, long before a student is expected to write and solve an equation in the problem setting above to answer the question, "If an employee's new salary is $24,610, what was her salary before the raise?" the student solves that problem experimentally:

Present salary ($)	12,000	15,500	22,000		
New salary ($)				24,610	34,240

* Calculators require students to pay attention to arithmetic ideas that otherwise would be of little interest to them.

Students must understand order of operations to use calculators. Division by 0 produces an ERROR display. Fractions are quotients. The subtraction key is different from the key that indicates a negative number. There is a square root key, a reciprocal key, a percent key. Scientific notation is necessary for displaying large numbers. Questions of how to recover a whole number remainder when one whole number is divided by another, how to use the calculator to raise a negative number to a power or to evaluate a polynomial at various values of the variable -- these are natural questions that require understanding of arithmetic properties to answer.

In addition students are motivated to use their calculators efficiently, and they seek ways to shorten keying sequences. The distributive property is central in establishing algorithms for evaluating polynomials and also in simple computations: $12{,}000 + .07(12{,}000) = 12{,}000(1.07)$. Concepts of algebra such as factoring can be anticipated in efforts to make numerical computation efficient. Some arithmetic ideas that make sense with paper and pencil computations (like rationalizing denominators) are not needed and need not be taught when calculators are used.

* Calculators permit the early introduction of graphing.

In a numerical approach to algebra we choose problem contexts in which the relationship between quantities in the problem describes a function. These problem contexts can be investigated numerically through chart-building and can also be displayed geometrically through graphs. It is important initially that

students plot large numbers of points in drawing a graph. Level 5 math students usually do not view a graph as a collection of points, but rather as a few points that have been connected. Calculators make it possible for students to plot so many points they they realize if they were able to plot all, they would get the complete graph.

Graphing gives a visual representation of functions; students can handle graphs without the vocabulary of functions. Our students even graph calculator keys: $\boxed{1/x}$, $\boxed{\sqrt{x}}$, $\boxed{x^2}$. Again, important ideas can be experienced in concrete cases.

Graphing strengthens a student's sense of number magnitude and order. It is a natural setting for confronting problems of scaling and of ratio and proportion. This type of arithmetic experience, which can be artificial if done for its own sake, is a natural part of graphing.

Graphing provides an additional problem solving strategy for students. The same problem settings that students deal with numerically can also be graphed. Once a graph is drawn, any number of questions can be answered -- including questions about inequalities. Students can solve equations and inequalities graphically before they learn techniques for solving them algebraically. They can, in fact, solve graphically (or numerically, for that matter) many problems for which they lack algebraic methods of solution. Calculators are important not only in drawing the graphs, but also in refining the estimates that are read from a graph.

Graphing can provide a bridge between numerical work and the formalism of algebra. Typically in these courses, problem situations are first investigated numerically, then revisited geometrically through graphs, and still later represented algebraically. For example, the problem about diluting an alcohol

solution stated numerically on page 6 has geometric and algebraic formulations at later times:

> Geometric: Water is added to 40 gallons of a 25% alcohol solution. Draw a graph that shows the relationship between the amount of water added and the concentration of the solution. (Use amounts of water from 0 to 80 gallons.) When is the concentration of alcohol equal to 12.5%? When is the concentration less than 10%?
>
> Algebraic: Water is added to 40 gallons of a 25% alcohol solution. Let x denote the number of gallons of water added; write an algebraic expression for the concentration of the new solution. How many gallons of water should be added to get a 12.5% solution?

Deciding what a variable represents and using it to describe the relationship in a problem are sophisticated procedures and difficult for many students. The numerical approach seems to guide the student in understanding the relationships in a problem, and the graph provides a visual representation. Students see that the numerical table, the graph, and the equation are three ways of describing the same relationships within a problem.

Graphing can both suggest and support more advanced concepts of algebra and can be very helpful to students who do not handle abstraction well. Quadratic inequalities are questions about where a parabola is above or below the axis, completing the square is a process that yields the coordinates of the vertex of a parabola, the domain of a function consists of the numbers that give points on its graph, and so on.

A Next Step

A great deal of what we do in our courses for remedial students can well be considered pre-algebra. We are troubled that the transition from arithmetic

to algebra is not being made successfully by large numbers of students in grades 8 and 9. In January, 1984, we began a project funded by SOHIO that will provide supplements for grades 7 and 8 in hopes of strengthening the pre-algebra component in these grades. The specific goal is to establish the concepts of function and variable before students move into algebra. Handheld calculators will play a central role.

Through several years of experience we believe we have come to understand ways in which calculators can give students entrance to many concepts of algebra and geometry. We have not had equivalent experience in using computers in the teaching of elementary mathematics. We hope to have an opportunity for some experimentation next year with students similar to those we have been teaching with calculators. Computers offer additional features, though it is not immediately clear how to exploit them in the teaching of algebra to a remedial audience. For example, we find that filling in several rows of a numerical chart makes generalizing with a variable very natural for students. Can the same experience be provided with a computer program? Can, in fact, a student handle programming without already appreciating the role of a variable? We have found that plotting many points to develop a graph strengthens students number intuition and sense of number order. Will this happen if computer graphics replace calculator and graph paper? Students can begin using calculators with essentially no instruction. Will providing students with computers require us to sacrifice mathematics instruction time for programming instruction? These are interesting questions and there are many others.

Undoubtedly algebra must remain in the curriculum of two-year colleges. Courses in mathematics and in many other areas require algebra. For the foreseeable future we will continue to teach many recent high school graduates

who do not know enough algebra to use it and also large numbers of adult students who have lost algebra skills they once may have had. We can use calculators (and maybe computers) to approach this mathematics in a concrete way that is sensible to students and focuses on the solving of problems. I hope to see separate beginning college courses in arithmetic, elementary algebra, and intermediate algebra give way to courses that integrate arithmetic and algebra and make full use of geometry. The precise content of those courses may not need to be the same for all students. Two-year colleges are excellent laboratories for seeking new approaches that will enable more students to learn more mathematics.

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